# Entrainment and Extraction Efficiency of Mixer-Settlers

C. A. SLEICHER, JR.

Shell Development Company, Emeryville, California

In mixer-settler extraction systems the settlers are seldom completely effective. Usually there occurs some entrainment of light phase in the heavy one and of heavy phase in the light one. This entrainment is analogous to axial mixing in continuous columns, which has been treated in the literature (3, 4). In this note a theoretical analysis is made of the effect of entrainment on the efficiency of countercurrent extractors consisting of multiple mixer-settler stages with finite rates of mass transfer.

The effect of entrainment on efficiency of extractors has been analyzed by Felix and Holder (2) for the case of mixers which operate at 100% efficiency, that is, infinite mass transfer rate. Their results however are unrealistic since their Equation (21) yields zero extraction for infinite internal entrainment rate. Such an extractor is actually equivalent to one stage. The difficulty can be traced to their boundary condition, which should be replaced by a balance around the first stage.

The present problem is closely related to that of the effect of entrainment on the efficiency of plate absorption towers and distillation columns, which was investigated in the 1930's by several authors (*1*). The problem of entrainment in extractors differs principally in that there is entrainment of two phases rather than one. Furthermore the solution presented here is for the effect of entrainment on the overall efficiency of extractors, whereas the distillation work concerned the efficiency of individual plates.

C. A. Sleicher, Jr., is now with the University of Washington, Seattle, Washington.

## DESCRIPTION OF EXTRACTOR MODELS

Figure 1 shows the extractor schematically and identifies symbols. The liquids leaving the first and last settlers go to after settlers or to purification equipment of some kind which returns entrained liquid to streams entering the mixers. Solute is transferred from a raffinate phase that passes countercurrent to the solvent phase. Each of the phases from a settler is contaminated with entrainment of the other phase. Note that $f$ is expressed as a fraction of $Q_f$; similarly $s$ is a fraction of $Q_s$. In this note "entrainment", when used quantitatively, refers to $f$ and $s$ and not to entrainment holdups.

From each settler at very high entrainment rates the concentrations of solute in the solvent leaving in the solvent phase and leaving as entrainment in the raffinate phase from each settler would be about the same. Similarly the concentration of solute in the raffinate phase entrained with the solvent stream would be equal to that in the raffinate leaving in the raffinate stream. At very low rates of entrainment however the entrained drops would be small and nearly in equilibrium with the entraining phase. Because of this difference calculations have been made for two models. The model for high entrainment rate is called *Model 1*, and the principal assumptions for it are the following:

1. The amount of entrainment in a given phase is the same from each settler.
2. The solvent and solute-free raffinate are immiscible (or their solubility does not vary with solute concentration and hence stage number).
3. The volume rates of solvent and feed phases do not change from stage to stage.
4. The distribution coefficient (equilibrium ratio) is constant; that is it is not a function of concentration.
5. Coalescence and redispersion of the dispersed phase and mixing of the continuous phase in the mixer is so rapid that the solute concentration is uniform throughout each phase.
6. The rate of mass transfer in the mixer is given by $Kav\ (x_n - my_n)$. $Kav$ is assumed to be the same for every mixer.
7. All mass transfer takes place in the mixer.

For Model 2 all assumptions are the same as for Model 1 except assumption 7. At the low rates of entrainment ascribed to Model 2 enough mass transfer is assumed in the settlers to allow the drops to reach equilibrium with the entraining phase. For this model, then, assumption 7 is the entrainment that leaves with each bulk phase from the settler is in equilibrium with that bulk phase.

This condition means that some mass transfer takes place in the settlers and that therefore the composition of the feed that leaves as entrainment with the solvent stream is different from the bulk concentration of the feed or raffinate stream from any settler.

## THE ANALYSIS AND ITS RESULTS

The extractor models can be described by the difference equations formed from appropriate material balances. These equations and their end conditions can be written in terms of the dimensionless variables $f$, $s$, $N$, $F$, $t$, $\Psi_n$, and $\Gamma_n$.

As in reference 4 the solution is expressed in terms of $\Psi_n$. It varies from 0 at the feed inlet stream to a possible maximum at the outlet of 1, which occurs when the raffinate is in equilibrium with the incoming solvent.

The solution of the difference equation for Model 1 is

$$\Psi_n = C_0 + C_1\lambda_1^n + C_2\lambda_2^n + C_3\lambda_3^n \quad (1)$$

where the $\lambda$ are the roots of

$$f(1+s)\lambda^3 - [(1+s)(1+t) + f(2+3s+Ft)]\lambda^2 + [(1+f)(1+Ft) + s(2+3f+t)]\lambda = s(1+f) \quad (2)$$

TABLE 1. REPRESENTATIVE RESULTS OF MODEL 1

$N = 6$

| $F$ | $f$ | $s$ | $t$ | $\psi_N/\psi_{NO}$ | $E_e$ | $E_o$ |
|---|---|---|---|---|---|---|
| 0.25 | 0.1 | 0.1 | 1 | 0.9926 | 0.95 | 0.32 |
| 0.25 | 0.1 | 0.1 | 10 | 0.9989 | 0.87 | 0.72 |
| 0.25 | 0.1 | 0.1 | ∞ | 0.9994 | 0.83 | 0.83 |
| 0.25 | 0.1 | 0.4 | ∞ | 0.9958 | 0.62 | 0.62 |
| 0.25 | 0.4 | 0.1 | ∞ | 0.9988 | 0.76 | 0.76 |
| 0.25 | 0.4 | 0.4 | ∞ | 0.9943 | 0.58 | 0.58 |
| 1 | 0.1 | 0.1 | 1 | 0.980 | 0.95 | 0.47 |
| 1 | 0.1 | 0.1 | 10 | 0.979 | 0.87 | 0.80 |
| 1 | 0.1 | 0.1 | ∞ | 0.978 | 0.86 | 0.86 |
| 1 | 0.1 | 0.4 | ∞ | 0.948 | 0.72 | 0.72 |
| 1 | 0.4 | 0.4 | ∞ | 0.923 | 0.63 | 0.63 |

The four constants of Equation (1) are determined from four end conditions, which are found by writing material balances for each phase at each end of the column. In writing these end conditions the after settlers are designated 0 and $N+1$. Observe that from the viewpoint of Mixer 1 the influent feed has composition $x_0$, and for Mixer $N$ the influent solvent has composition $y_{N+1}$. The end conditions are then

| Dimensional form | Dimensionless form | |
|---|---|---|
| $x_N = x_{N+1}$ | $\Psi_N = \Psi_{N+1}$ | (3) |
| $x_1 + fx_1 = (1+f)x_0$ | $f\Psi_1 = (1+f)\Psi_0$ | (4) |
| $y_0 = y_1$ | $\Gamma_0 = \Gamma_1$ | (5) |
| $y_1 + sy_N = (1+s)y_{N+1}$ | $s\Gamma_N = (1+s)\Gamma_{N+1}$ | (6) |

Equations (3) and (5) specify that no mass transfer takes place in the after settlers, and Equations (4) and (6) specify the composition of the feed to stage 1 and of the solvent to stage $N$, respectively.

Substitution of $\Psi$ from Equation (1) into Equations (3) through (6) yields four linear algebraic equations that can be solved for the constants $C_m$. These equations will be designated by the numbers (7) through (10).

For special cases the solutions are much simpler than the general solution represented by Equations (1), (7), (8), (9), and (10). Listed below for reference are solutions for particular cases of interest.

For equilibrium in the mixer

$$\Psi_n = \frac{F\lambda^n - \lambda}{F^2\lambda^N - \lambda} \tag{11}$$

where $\lambda = (F + fF + s)/(1 + fF + s)$ and when $F = 1$

$$\Psi_n = \frac{f + s + n}{2(f + s) + 1 + N} \tag{12}$$

With no entrainment

$$\Psi_n = \frac{\lambda^n - 1}{F\lambda^N - 1} \tag{13}$$

where $\lambda = (1 + Ft)/(1 + t)$ and when $F = 1$,

$$\Psi_n = \frac{tn}{1 + t + tN} \tag{14}$$

With no entrainment and equilibrium in the mixer

$$\Psi_n = \frac{F^n - 1}{F^{N+1} - 1} \tag{15}$$

and when $F = 1$,

$$\Psi_n = \frac{n}{1 + N} \tag{16}$$

For infinite entrainment of both phases

$$\Psi_1 = \Psi_2 = \ldots = \Psi_N = \frac{Nt}{1 + Nt + FNt} \tag{17}$$

The equations for the general case were solved on a digital computer for about 800 combinations of the five parameters in the following range: $3 \leq N \leq 10$, $0.25 < F < 4$, $0.3 < t < \infty$, $0 < f$ and $s < 0.8$. Table 1 gives some representative results. The settler efficiency $E_e$ is defined by $N_o/N$, where both $N_o$ and $N$ reflect the same $t$, $F$, and $\Psi_N$. It was found that the settler efficiency is only weakly dependent on $N$ and $F$ and is more strongly dependent on $t$ at low $F$. The magnitude of $E_e$ is of particular interest. For example with $F = 1$ and $N = 6$, 10% entrainment of each phase yields a settler efficiency of 0.86 to 0.95, and 20% entrainment yields 0.77 to 0.89. These entrainment rates are quite high, and yet the efficiency is decreased by at most 23%. It is not until the entrainment of each phase reaches 0.4 that the stage efficiency falls below 60%. The over-all efficiency is of course strongly dependent on the rate of mass transfer (expressed by $t$) as well as entrainment. Throughout most of the studied range of parameters the settler efficiency is correlated within 5% by the following empirical equation:

$$E_e = \frac{\log F + (N - 1)\log \lambda'}{N \log F} \tag{18}$$

where

$$\lambda' = \frac{F + E_m(Ff + s)}{1 + E_m(Ff + s)}$$

For $F = 1$ this simplifies to

$$E_e = \frac{N + E_m(f + s)}{N + NE_m(f + s)}$$

Here $E_m$ is the efficiency of a single mixer, $Ft/(1 + Ft)$. It is easily shown that this expression for the mixer efficiency is equivalent to the Murphree plate efficiency of a distillation column. Equation (18) is exact for the case of $t = \infty$ and is proposed in place of Equation (22) of reference 2.

For the over-all efficiency it was found that multiplying the settler efficiency by an over-all mixer efficiency gives an adequate correlation. The over-all efficiency of an extractor with no entrainment is equal to the mixer efficiency only when the extraction factor is unity. In general

$$E_{mo} = \frac{\log \dfrac{1 + Ft}{1 + t}}{\log F} \tag{19}$$

This equation multiplied by the settler efficiency correlates the over-all efficiency within 10% throughout most of range of parameters investigated; that is

$$E_o = E_e E_{mo} \tag{20}$$

Equations (18), (19), and (20) afford a means of calculating the number of actual stages of a mixer-settler extractor from the entrainment and mass transfer rates.

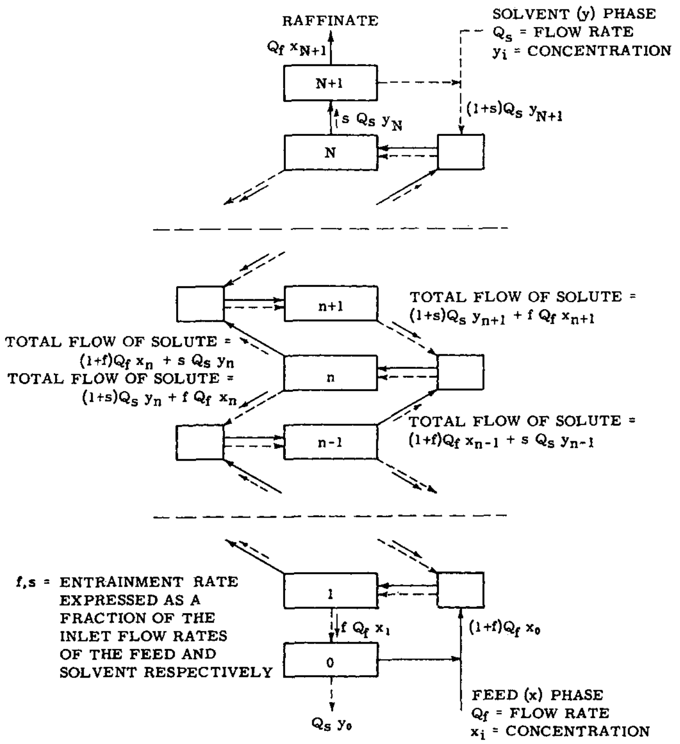


Fig. 1. Nomenclature for staged extraction, Model 1.

#### Model 2

The difference equation for Model 2 is of second rather than fourth order. Its solution is

$$\Psi_n = \frac{FH\lambda^n - J}{F^2H\lambda^N - J}$$

where

$$H = 1 + fF + s$$

$$J = F + fF + s$$

$$\lambda = \frac{H[F(1+f)(1+f+s)+J(Ft+fFt+st+s+2fs)]}{J[t(1+fF+s)^2+f^2F(1+2s)+(1+s)^2(1+2f)]}$$

Only a few calculations were made for Model 2 because it became evident that at entrainment rates in the range of validity of the model (probably $f$ and $s$ of 0.1 or less) the entrainment has little effect on extraction. When equilibrium is reached in the mixer, both models give the same result. For other cases the extent of extraction is always greater for Model 2 because of the mass transfer in the settler.

## USE OF THE ANALYSIS AND CONCLUSIONS

This analysis will find utility in the design of settlers, for it suggests that settlers other than after settlers are sometimes over-designed. If there are no complicating consequences of entrainment, such as the formation of stable emulsions, entrainment rates as high as 10% are rarely consequential.

The results will also be useful for interpreting experimental data. In particular if an extractor has low efficiency, measurements of entrainment from the settlers will establish whether the low efficiency is caused by the entrainment or by poor mass transfer in the mixers.

The major conclusion to be drawn from this work is that high entrainment rates have relatively little effect on the efficiency of mixer-settler extractors. When such extractors have poor efficiency, it is therefore more likely to be caused by low mass transfer rate than by entrainment from the settlers. This suggests that efficiency of some mixer settlers might be improved by operating the mixers at higher rates provided that (1) there occurs no deleterious effect such as the formation of stable emulsions and (2) the capacity of the after settlers is adequate. If after settlers are not used, then improvement might be obtained without excessive entrainment in the effluents by increasing the mixing rates in all but the first and last mixers.

## NOTATIONS

$E_e$ = entrainment or settler efficiency, $N_o/N$
$E_m$ = Murphree efficiency in distillation or mixer efficiency in extraction, $Ft/(1 + Ft)$
$E_{mo}$ = over-all efficiency of a column with no entrainment [see Equation (19)]
$E_o$ = over-all efficiency, $N_t/N$
$F$ = extraction factor, $mQ_f/Q_s$
$f$ = fraction of $Q_f$ entrained in the solvent phase from a settler
$Kav$ = product of a mass transfer coefficient and the interfacial area per mixer
$N$ = number of actual stages
$N_o$ = number of stages to effect a given extraction with no entrainment
$N_t$ = number of theoretical stages
$Q_f$ = flow rate of feed phase to extractor
$Q_s$ = flow rate of solvent phase to extractor
$s$ = fraction of $Q_s$ entrained in the feed phase from a settler
$t$ = $Kav/Q_s$
$x$ = concentration of solute in feed or raffinate phase
$y$ = concentration of solute in solvent phase
$\Psi_n$ = dimensionless concentration of feed phase, $(x_i - x_n)/(x_i - my_i)$
$\Psi_N/\Psi_{NO}$ = $\Psi(s, f, F, t, N)/\Psi(0, 0, F, t, N)$
$\Gamma_n$ = dimensionless concentration of solvent phase, $(y_n - y_i) Q_s/(x_i - my_i)Q_f$

## LITERATURE CITED


1. Colburn, A. P., *Ind. Eng. Chem.*, **28**, 526 (1936).
2. Felix, J. R. and C. H. Holder, *A.I.Ch.E. Journal*, **1**, 296 (1955).
3. Miyauchi, T., *USAEC*, UCRL-3911 (1957).
4. Sleicher, C. A., Jr., *A.I.Ch.E. Journal*, **5**, 145 (1959).


---

# CHEMICAL ENGINEERING PROGRESS SYMPOSIUM SERIES ABSTRACTS

**The Chemical Engineering Progress Symposium Series is composed of papers on specific subjects conveniently bound in individual books, which are published at intervals. The books are 8½ by 11 inches, paper covered, and cost $3.50 to members, $4.50 to nonmembers. They may be ordered from the Secretary's Office, the American Institute of Chemical Engineers, 25 West 45 Street, New York 36, New York.**

**The *A. I. Ch. E. Journal* will publish, from time to time, abstracts of the articles appearing in the Symposium Series volumes. Recently published volumes are abstracted below.**

### CHEMICAL ENGINEERING EDUCATION—ACADEMIC AND INDUSTRIAL, Vol. 55, No. 26, 1959.

**Evolution of the Unit Operations,** Warren K. Lewis. The technological achievements of the profession in America during the last half century have been impressive. No small part of the credit is due to the training the young engineer has received. The emphasis on basic science has given the understanding of inner mechanisms and the analytical ability to foresee and plan for ultimate results. The unit operations approach has stimulated technical imagination and has shown the way to the transfer of experience acquired in one area to others which sometimes seemed unrelated. Moreover, making it possible to reduce coverage of detail without sacrifice in breadth has resulted in training in greater depth. The foundations laid by the educational pioneers have proved sound and dependable. **Development of Chemical Engineering at the University of Michigan,** Donald L. Katz. The sixty years of chemical engineering education are surveyed for the influences that have dominated the curriculum. As in most schools, at Michigan the birth of this new branch of engineering was due largely to teachers of chemistry. The association of the faculty with industrial processes drew the teacher into research on engineering problems and provided the stimulus for new engineering courses. The evolution of the